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AIASYB-2
Aplicación de la Inteligencia Artificial a los Sensores y Biosensores

FUZZY LOGIC

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OUTLINE

- **UNCERTAINTY**
- **FUZZY LOGIC**
- **FOUNDATIONS**
  - **FUZZY SETS**
- **FUZZY SYSTEM**
- **EXAMPLE**

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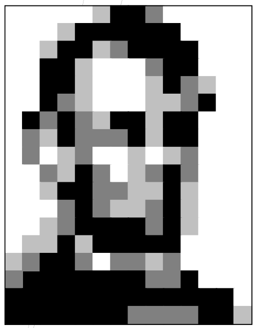
TYPES OF UNCERTAINTY

- IMPRECISION, VAGUENESS, UNCERTAINTY, …
  - **Stochastic**
    - Statistical models
      - The probability of hitting the target is 0.8
  - **Non statistics (Lexical)**
    - Fuzzy Models
      - Tall Men, Hot Days, Successful Year



Useful information

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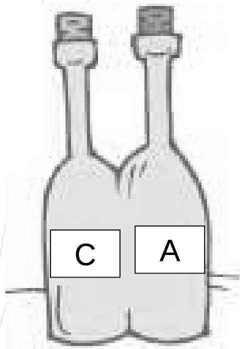
MODELS OF UNCERTAINTY

- R = {all the liquids}
- L = {potable liquids (drinkable)}



Masked bottles

$$Pr(A \in L) = 0.91$$

$$m_L(C) = 0.91$$

Bezdec

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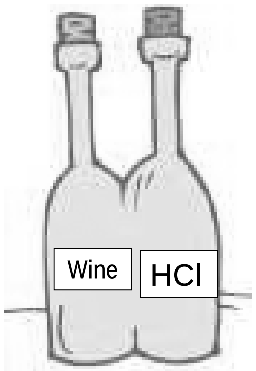
PROBABILITY ≠ FUZZINES

The pair of bottles unmasked

- PROBABILITY
  - Number of ocurrences (frequency)
    - Probability is only valid for future/unknown events
- FUZZINES
  - Similarity
    - Fuzzy set membership continues after the event



$$m_L(C) = 0.91$$
$$Pr(A \in L) = 0$$

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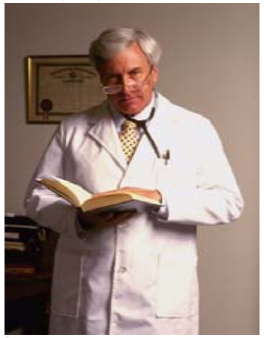
Probability and Uncertainty

- "... a person suffering from hepatitis shows in 60% of all cases a strong fever, in 45% of all cases yellowish colored skin, and in 30% of all cases suffers from nausea ..."



Stochastics and Fuzzy Logic Complement Each Other !

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FUZZY LOGIC

- ARTIFICIAL INTELLIGENCE (Soft Computing)
  - Human reasoning
- SIMPLE AND POWERFUL
- EXTENDED (Applications)
- RESEARCHING

It is not always the best

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FUZZY LOGIC

- Reasoning with uncertainty (humanlike)
  - Common sense , approximate reasoning
- Vagueness in the definition (linguistic terms)
- Heuristic, qualitative approach
- Multi-valued logic: *linguistic true values*
  - It includes other type of logics (classical)
    - Aristotle (350 BC)
  - Lukasiewicz
    - 3-valued logic in 1920
    - many-valued logic in 1923 (degrees of truth)

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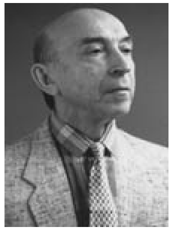
ORIGIN

Lofti Zadeh, 1921



◆ "Fuzzy Sets" paper published in 1965

- Fuzziness: "A type of imprecision which is associated with classes in which there is no sharp transition from membership to non-membership" - Zadeh (1970)

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WHEN FUZZY LOGIC IS USEFUL?

Depends on the type of the available information

a) Complex processes
b) Difficult to estimate the parameters
c) Difficult to get accurate measurements from sensors
e) Lack of reliability of the sensors
f) Noisy environment,

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APPLICATIONS

◆ Pattern recognition, clustering, image processing, diagnosis, etc.
◆ Biological, medical processes, quality control
◆ Complex systems: high speed trains, helicopters, automatic driving, lifts, boats, washing machines
◆ Control applications
◆ ….

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History, State of the Art, and Development

1965	Seminal Paper "Fuzzy Logic" by Prof. Lotfi Zadeh, Faculty in Electrical Engineering, U.C. Berkeley, sets the Foundation of the "Fuzzy Set Theory"
1970	First Application of Fuzzy Logic in Control Engineering (Europe)
1975	Introduction of Fuzzy Logic in Japan
1980	Empirical Verification of Fuzzy Logic in Europe
1985	Broad Application of Fuzzy Logic in Japan
1990	Broad Application of Fuzzy Logic in Europe
1995	Broad Application of Fuzzy Logic in the U.S.
2000	Fuzzy Logic becomes a standard technology and is also applied in Data and Sensor Signal Analysis, Business and Finance.

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FOUNDATIONS

- numbers much bigger that 100,
- long rivers,
- young people,
- sunny days,
- difficult paragraph, etc.

Transition between membership and non-membership is not sharp but gradual

Assigned numbers to objects based on the degree to which it was perceived to belong to the class

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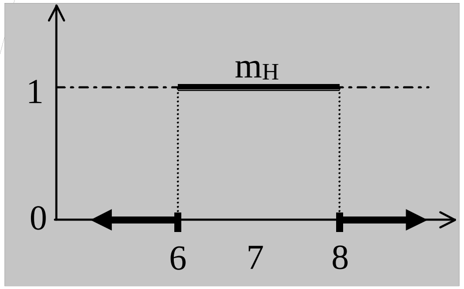
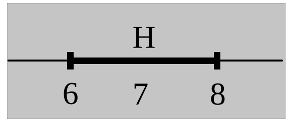
CRISP SETS

◆ Number between 6 and 8

- $m_H(r)=\{0,1\}$

$$m_H(r) = \begin{cases} 1, & 6 \le r \le 8 \\ 0, & \text{otherwise} \end{cases}$$





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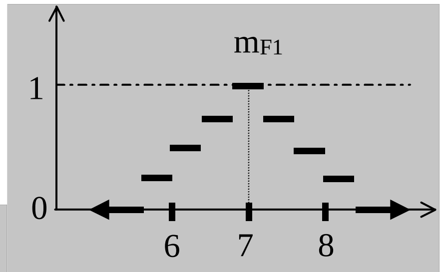
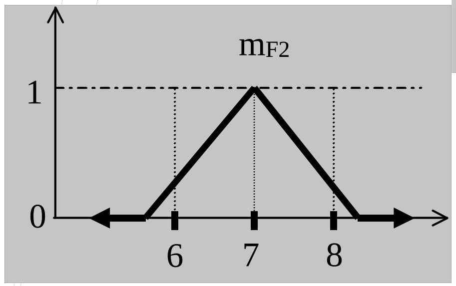
FUZZY SETS

- Near 7
- $m_F(r) = [0,1]$





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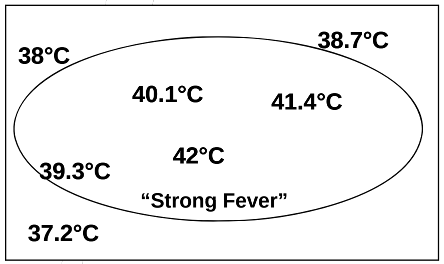
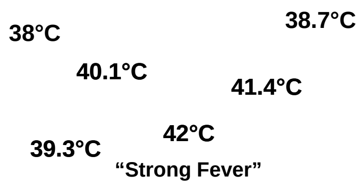
FUZZY SET THEORY



Conventional (boolean) Set Theory

Fuzzy Set theory



"More-or-Less" Rather Than "Either-Or" !

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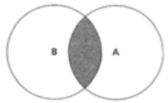
MEMBERSHIP FUNCTION

- A set is defined by the characteristic or membership function
  - Odd numbers



A point is either *in* the set or *not;* it's either *in* the intersection or *not.*

- *Partial belonging to a set* (L = [0,1])
  - Old people $\mu_{\underset{\sim}{F}} : U \rightarrow [0,1]$

Membership function can be defined analytically, graphically, by a vector, by a function

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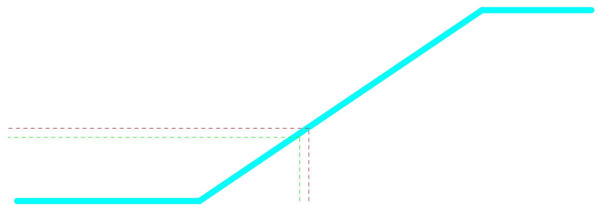
MEMBERSHIP FUNCTION

Discrete Definition:

μ_{SF}(35°C) = 0 **μ_{SF}(38°C) = 0.1** **μ_{SF}(41°C) = 0.9**
μ_{SF}(36°C) = 0 **μ_{SF}(39°C) = 0.35** **μ_{SF}(42°C) = 1**
μ_{SF}(37°C) = 0 **μ_{SF}(40°C) = 0.65** **μ_{SF}(43°C) = 1**

Continuous Definition:

No More Artificial Thresholds!



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FUZZY SET

Given a *Universe of Discourse U* (domain), a fuzzy set **A** of U is a set of ordered pairs of elements x and the corresponding membership degree to the set

A = {(*x* | A(*x*)}

Elements of the universe of discourse *U*, can belong to the fuzzy set A with any value between 0 and 1

$0 \leq \mu_{\underset{\sim}{F}}(u) \leq 1$ Degree of membership

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PROPERTIES OF FUZZY SETS

- PROPERTIES
  - Normalized (m_F(7) = 1: true)
  - Symmetry
- USEFUL INFORMATION
  - $m_H(q) = 1$
  - $m_F(q) = 0.98$
- FLEXIBILITY
  - Include imprecision

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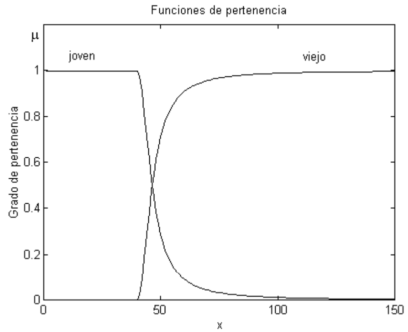
CONTINUOUS FUZZY SET

- U: age
  - Fuzzy set young **J**
    - **J** = {(x | 1), 0 < x < 40; (x | (1 + (x - 40)^2 / 40)-1), x > 40}
  - Fuzzy set old **V**
    - **V** = {(x | 0), 0 < x < 40; (x | (1+ 40 / (x - 40)^2)-1), x > 40}



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DICRETE FUZZY SET

- U: { months of the year}
  - Fuzzy set Cold **F**

F = 1 | January + 1 | February + 0.8 | March + 0.7 | April + 0.5 | May + ...

  - Fuzzy set Hot **C**

C = 0 | January + 0 | February + 0.4 | March + 0.6 | April + 0.6 | May + ...

U	*Ene.*	*Feb.*	*Mar*	*Abr.*	*May*	*Jun.*	*Jul.*	*Ago*	*Sept*	*Oct.*	*Nov*	*Dic.*
Frío	1	1	0.8	0.7	0.5	0.4	0.2	0	0.3	0.5	0.8	1
Calor	0	0	0.4	0.6	0.6	0.8	1	1	0.7	0.4	0.1	0

Semantic values of the fuzzy sets (young, old, cold, ...) = <u>linguistic labels</u>

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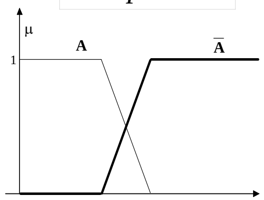
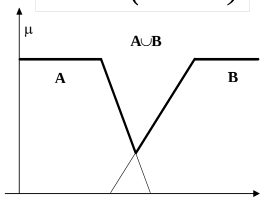
FUZZY SET OPERATIONS

Complement



$$\mu_{\underset{\sim}{C}}(x) = 1 - \mu_{\underset{\sim}{A}}(x)$$

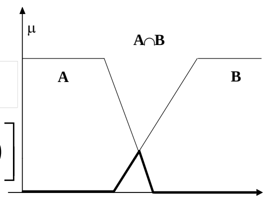
Union (s-norm)



$$\mu_{\underset{\sim}{C}}(x) = \max\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\right]$$

Intersetion (t-norm)

$$\mu_{\underset{\sim}{C}}(x) = \min\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\right]$$



Not - and - or

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PROPERTIES OF SET OPERATIONS

Most of the properties that hold for classical sets (e.g., commutativity, associativity and idempotence) hold also for fuzzy sets except for following two properties:

Law of contradiction $\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} \neq \phi$

The intersection of a fuzzy set and its complement results in a fuzzy set with membership values of up to ½ and thus does not equal the empty set (as in the case of classical sets)

Law of excluded middle $\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq U$

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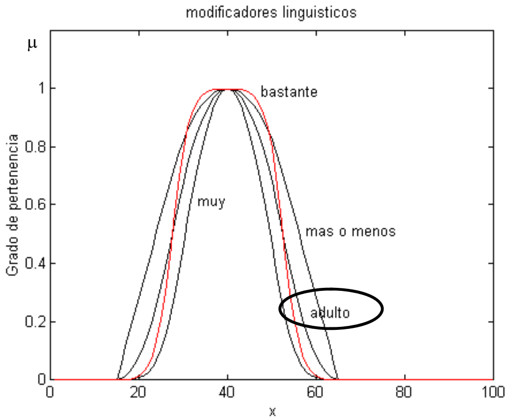
LINGUISTIC MODIFIERS (HEDGES)



Adverbs

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KINDS OF MODIFIERS

- Intensify a fuzzy set (very, extremely)
  - "Very" can be the mathematical square
- Dilute a fuzzy set (somewhat, sort of)
  - "Somewhat" can be the square root
- Express probabilities (probably, not likely)
- Approximate a scalar or single number (exactly)
- Express vague quantities (most, seldom)

Other conventions are possible

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FUZZY RELATIONS

- CRISP RELATIONS BETWEEN FUZZY SETS
  - If there exits any relation between the elements of two or more sets
  - Example: Equivalence, inclusion

Two fuzzy sets are equal if and only if *all* elements have identical membership values

- FUZZY RELATIONS BETWEEN CRISP SETS
  - Membership degree to the relation (between 0 and 1)
- FUZZY RELATIONS BETWEEN FUZZY SETS

$$\mathbf{R} \subset \mathbf{A} \times \mathbf{B}$$

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HOW TO REPRESENT THE KNOWLEDGE

- REPRESENTATION OF THE EXPERT KNOWLEDGE
  - LINGUISTIC VARIABLES
    - Temperature = {very high, high, medium, low, ...}
  - FUZZY RULES
    - If the temperature is high then ...

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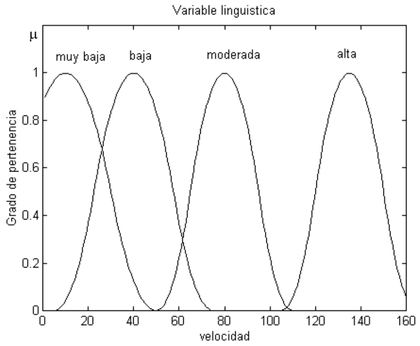
LINGUISTIC VARIABLE

"a variable whose values are words or sentences in a natural or artificial language." – Zadeh

- Variable name
- Universe or domain
- Linguistic labels (**terms**)
  - Semantic
- Fuzzy set assigned to each label (membership function)
- Degree of memebership



Linguistic Variable *speed*

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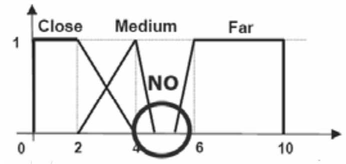
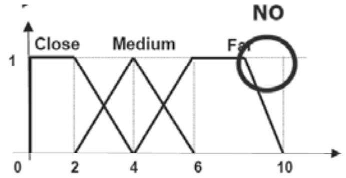
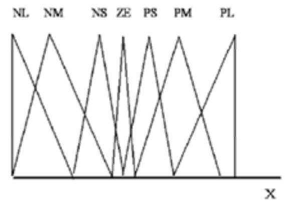
MEMBERSHIP SELECTION

- ✓ **NUMBER (Granularity: from 3 to 7)**
- ✓ **COVERAGE (any point at least by one fuzzy set)**
- ✓ **BOUNDARIES (covered with maximum value)**
- ✓ **CROSSPOINT**







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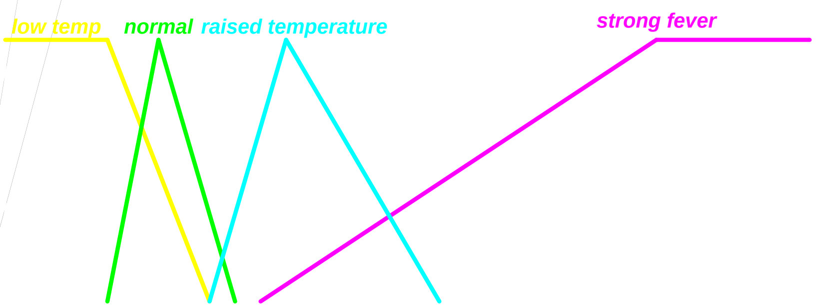
EXAMPLE: LINGUSTIC VARIABLE

> Can be any shape, including arbitrary or irregular
➢Is normalized to values between 0 and 1
➢ Often uses triangular approximations to save computation time



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FUZZY REASONING

- ◆ FORWARD CHAINING
  - ONE LEVEL
  - GMP (MODUNS PONES GENERALIZED)
  - CRI (COMPOSITIONAL RULE OF INFERENCE)
    - MAMDANI: max-min
    - LARSEN: max-prod

FUZZY REASONING

GMP but the first premise is only one variable

premise 1: *x* is A’
premise 2: (R) if *x* is A then *y* is B

---

conclussion: *y* is B’

$$R = (A(x) \rightarrow B(y))$$

$$B' = A' \circ R = A' \circ (A \times B)$$

$$\mu A' \circ (A \times B)(y) = \max [\mu(A'(x) * \mu A \times B(y))]$$

TYPES OF FUZZY SYSTEMS

- MAMDANI

  Ri: **if** *x* is Ai **and** *y* is Bi **then** *z* is Ui

- TAKAGI-SUGENO (TSK)

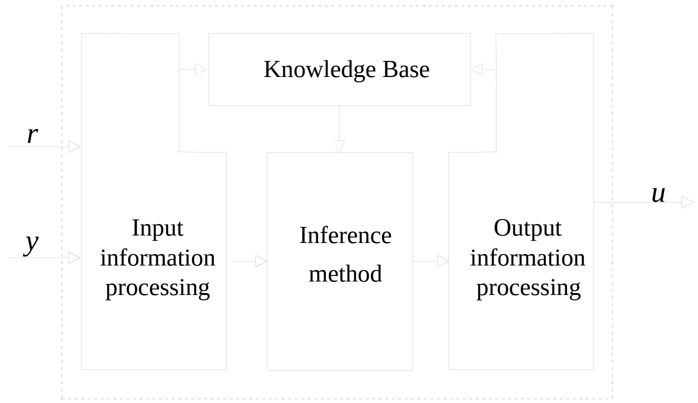
  Ri: **if** *x* is Ai **and** *y* is Bi **then** $z = fi(x,y)$

FUZZY SYSTEM: DEFINITION

A system which emulates a human expert. The expert knowledge is put in the form of a set of linguistic rules that have as antecedents the possible values of the inputs, and that conclude a linguistic action

```
If temperature is high, then lower much the heating
```


MODULES OF A FUZZY SYSTEM



Fuzzy system components

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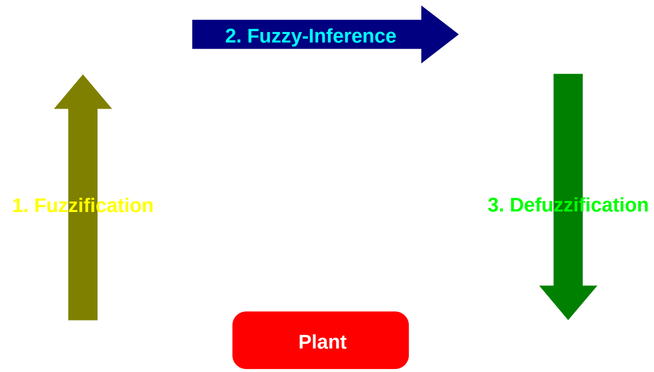
ELEMENTS OF A FUZZY LOGIC SYSTEM



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INPUT PROCESSING

- *Input variables: selection and measurement*
  - *From sensors, data or user*
  - *Computed from perceived variables (error, derivatives, composition of variables)*
- *Scaling*
- *Fuzzification*

Fuzzification depends on the input information

$$\boxed{A(x) = fuz(x_0) = A(x_0)}$$

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KNOWLEDGE BASE

- *Data Base*
  - Membership functions
  - Input and output grid
    - Number of labels: fuzzy sets
- *Rule Base:* **if ... then**

Number of input variables

		2	3	4	5
Nº labels	*2*	4	8	16	32
	3	9	27	108	324
	4	16	64	256	1024

Number of rules

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INFERENCE MECHANISM

- *Reasoning*

  *RCI max-**

$$z = y° (x° R)$$

R1: **if** *height* is tall **y** *weight* is big **then** *fatness* is medium

R2: **if** *height* is tall **y** *weight* is low **then** *fatness* is small

R3: **if** *height* is short **y** *weight* is big **then** *fatness* is big

R4: **if** *height* is short **y** *weight* is low **then** *fatness* is small

R = also(R1, R2, R3, …) = ∪(R1, R2, R3, …)

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FUZZY INFERENCE: RULES

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- Aggregation
  - Computing the "IF"-Part

Boolean Logic Only Defines Operators for 0/1:

A	B	AvB
0	0	0
0	1	0
1	0	0
1	1	1

Fuzzy Logic Delivers a Continuous Extension:

AND: $\mu_{AvB} = \min\{ \mu_A; \mu_B \}$

OR: $\mu_{A+B} = \max\{ \mu_A; \mu_B \}$

NOT: $\mu_{-A} = 1 - \mu_A$

- Composition
  - Computing the "THEN"-Part

OUTPUT PROCESSING

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◆ *Selection of the output variables*

◆ *Defuzzification*

Defuzzification: obtain a crisp value from a fuzzy output

$$\boxed{z_0 = defuz(z)}$$

$$z = \frac{\sum_{i=1}^{n} \omega_i . y_i}{\sum_{i=1}^{n} \omega_i}$$

◆ *linear defuzzification*

◆ *mean of maxima(MOM)*

◆ *center of area (COA)*

DEFUZZIFICATION

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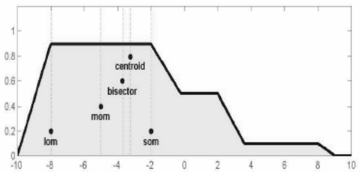
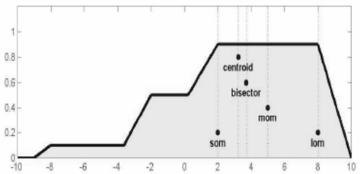
◆ Centroid
◆ Bisector
◆ Mean of maxima
◆ Lowest of maximum
◆ Sum of maxima
◆ Center of the highest
◆ ...





EXAMPLE

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Inputs:
- error e(t)
- change in error ce(t)

Output:
- control u(t)

$e(t)$, $ce(t)$ → **Fuzzy System** → $u(t)$

Universe of discourse [-Lx, Lx]

Linguistic Variables : error, change in error, control

Labels: P: positive
Z: zero
N: negative

Fuzzification strategy: Mamdani

Inference: max-min

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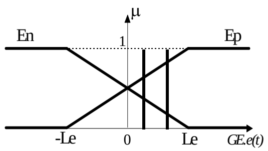
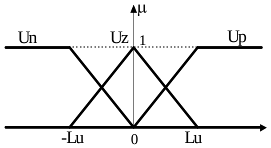
EXAMPLE

- Membership functions and fuzzification:





Ne = Nce = 2
Le = Lce = 10

Nu = 3
Lu = 10

e(t) = 3 $\Rightarrow$ 0.6 Ep, 0.4 En   ce(t) = 6 $\Rightarrow$ 0.8 CEp, 0.2 CEn

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EXAMPLE: rules

- R1: **if** *e* is P **and** *ce* is P **then** U is P
- R2: **if** *e* is P **and** *ce* is N **then** U is Z
- R3: **if** *e* is N **and** *ce* is P **then** U is Z
- R4: **if** *e* is N **and** *ce* is N **then** U is N

min

e(t)	**and**	ce(t)	U
0.6Ep		0.8CEp	0.6Up
0.6Ep		0.2CEn	0.2Uz
0.4En		0.8CEp	0.4Uz
0.4En		0.2CEn	0.2Un

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EXAMPLE: defuzzification

- Defuzzification

$$u_L(t) = \sum_{Nu} u_k p_k \quad \mathbf{= Up.Lu+Uz.0+Uz.0+Un(-Lu)}$$

$$u_{COA}(t) = \frac{\sum_{Nu} u_k p_k}{\sum_{Nu} u_k}$$

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